C. ABDUL HAKEEM COLLEGE (AUTONOMOUS),
MELVISHARAM - 632 509.
SEMESTER EXAMINATIONS, NOVEMBER - 2018

M.Sc., MATHEMATICS **SEMESTER I**

P18MMA102 – REAL ANALYSIS - I

Time: Three Hours Maximum: 75 Marks

SECTION - A (5 X 6 = 30 Marks)

Answer **ALL** Questions.

1. a) Let f be an increasing function defined on $[a,b]$ and let $x_0, x_1, \dots, x_n$ be $n+1$ points such that $a = x_0 < x_1 < \cdots < x_n = b$. Then prove that $\sum_{k=1}^{n-1}[f(x_k+) - f(x_k-)] \le f(b) - f(a)$.

(Or)

b) If f is continuous on $[a,b]$ and if f' exists and is bounded in the interior, say $|f'(x)| \le A$ for all $x \epsilon (a,b)$, then prove that f is of bounded variation on $[a,b]$.

2. a) If $f \epsilon R(\alpha)$ and if $g \epsilon R(\alpha)$ on $[a,b]$, then prove that $c_1 f + c_2 g \ \epsilon R(\alpha)$ on $[a,b]$, also show that $\int_a^b (c_1 f + c_2 g) d\alpha = c_1 \int_a^b f\, d\alpha + c_2 \int_a^b g\, d\alpha$.

(Or)

b) Assume that $\alpha \nearrow$ on $[a,b]$. Then prove that (i) If P' is finer than P then $U(P', f, \alpha) \le U(P, f, \alpha)$ and $L(P', f, \alpha) \ge L(P, f, \alpha)$, (ii) For any two partitions $P_1 and\ P_2$, $L(P_1, f, \alpha) \le U(P_2, f, \alpha)$.

3. a) If f is continuous on $[a,b]$ and if α is of bounded variation on $[a,b]$, then prove that $f \epsilon R(\alpha)$ on $[a,b]$.

(Or)

b) State and prove second Mean-Value Theorem for Riemann- Stieltjes integrals.

4. a) State and Prove Dirichlet's test.

(Or)

b) Assume that each $a_n > 0$. Then prove that the product $\prod(1 - a_n)$ converges if and only if, the series $\sum a_n$ converges.

5. a) Assume that $f_n \to f$ uniformly on S. If each f_n is continuous at a point c of S, then prove that the limit function f is also continuous at c.

(Or)

b) State and prove Cauchy condition for uniform convergence of series.

SECTION - B (3 X 15 = 45 Marks)

Answer **ANY THREE** Questions.

6. Let f be of bounded variation on $[a,b]$ and assume that $c \epsilon (a,b)$. Then prove that f is of bounded variation on $[a,c]$ and on $[c,b]$, also show that $V_f(a,b) = V_f(a,c) + V_f(c,b)$.

7. Assume that $c \epsilon (a,b)$. Prove that if two of the three integrals in $\int_a^c f d\alpha + \int_c^b f d\alpha = \int_a^b f\, d\alpha$ exists, then the third also exists.

8. Let α be of bounded variation on$[a,b]$. Let $V(x)$ denote the total variation of α on $[a,x]$ if $a < x \le b$, and let $V(a) = 0$. Let f be defined and bounded on $[a,b]$. If $f \epsilon R(\alpha)$ on $[a,b]$, then prove that $f \epsilon R(V)$ on $[a,b]$.

9. State and prove Mertens theorem.

10. Let α be of bounded variation on$[a, b]$. Assume that each term of the sequence $\{f_n\}$ is real –valued function such that $f_n \epsilon R(\alpha)$ on $[a, b]$, for each $n = 1,2, \ldots$ Assume that $f_n \to f$ uniformly on $[a, b]$ and define

$g_n(x) = \int_a^x f_n(t) d\alpha(t)$ if $x \epsilon [a, b], n = 1,2, \ldots$ Then prove that

(i) $f \epsilon R(\alpha)$ on $[a, b]$,

(ii) $g_n \to g$ uniformly on $[a, b]$, where $g(x) = \int_a^x f(t) d\alpha(t)$.

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